Momentum-Aware Planning Synthesis for Dynamic Legged Locomotion

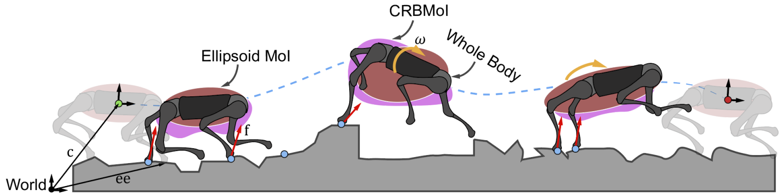
Ziyi Zhou, Bruce Wingo, Nathan Boyd, Seth Hutchinson, and Ye Zhao
Institute for Robotics and Intelligent Machines, Georgia Institute of Technology

Introduction and Objective

- Hierarchical gait-->centroidal-->whole-body pipelines reduce planning complexity, additional constraints on momentum and full-body kinematics enable more dynamically feasible solutions.
- **Design a centroidal optimization capable of discovering both contact sequences and angular momentum trajectories.**
- **Achieve a dynamic consensus between centroidal and whole-body models** using constrained ADMM.

Centroidal and Whole-Body Optimization

- Centroidal optimization utilizes a single rigid body model with equimomental-ellipsoid-based Moment of Inertia (MoI) [2, 3].
- Simultaneously **solve for footholds, contact forces, centroidal and momentum** trajectories.
- **Ellipsoid MoI tracks joint motion effects on Composite Rigid Body MoI** from whole-body model for accurate momentum generation.
- WBD tracks the consensus quantities from centroidal optimization, and then solved via Differential Dynamic Programming (DDP).



ADMM Constrained Trajectory Optimization

- The consensus [1] is enforced by adding equality **consistency constraints for Center of Mass (CoM) positions, momentum, footholds**. The MoI is directly computed from whole-body CRBMoI.

Sample for one iteration procedure:

Centroidal Primal Updates

$$\min_{\phi_{\text{cen}}} \sum_{k=1}^{K}\sum_{i=1}^{N_k} \mathcal{L}_{\text{cen}}(\phi_{\text{cen}}) + \sum_{i\in\mathcal{I}} \frac{\rho_i}{2}\|\mathbf{r}_i + \mathbf{w}_i^k\|_2^2$$

$$\text{s.t.} \quad \begin{bmatrix} m\ddot{\mathbf{c}} \\ \dot{\mathbf{h}} \end{bmatrix} = \begin{bmatrix} \sum_j \mathbf{f}_j + m\mathbf{g} \\ \sum_j \mathbf{f}_j \times (\mathbf{c} - \mathbf{ee}_j) \end{bmatrix}$$

$$\mathbf{h} = \mathbf{I}_{\text{ellip}}\boldsymbol{\omega}$$

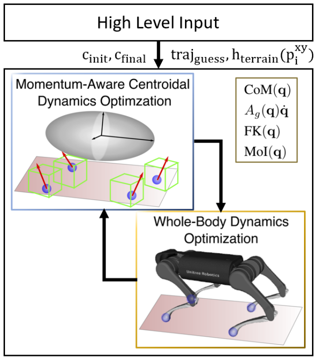
Force, terrain and RoM constraints

WBD Primal Updates

$$\min_{\phi_{\text{wbd}}} \sum_{k=1}^{K}\sum_{i=1}^{N_k} \mathcal{L}_{\text{wbd}}(\phi_{\text{wbd}}) + \sum_{i\in\mathcal{I}} \frac{\rho_i}{2}\|\mathbf{r}_i + \mathbf{w}_i^k\|_2^2$$

$$\text{s.t.} \quad \mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{B}\mathbf{u} + \mathbf{J}_c^T\boldsymbol{\lambda}_c$$

$$\ddot{\mathbf{x}}_e = \mathbf{J}_c\ddot{\mathbf{q}} + \dot{\mathbf{J}}_c\dot{\mathbf{q}} = 0$$



$$\mathbf{w}_c^{k+1} = \mathbf{w}_c^k + \text{CoM}(\mathbf{q}^{k+1}) - \mathbf{c}^{k+1}$$

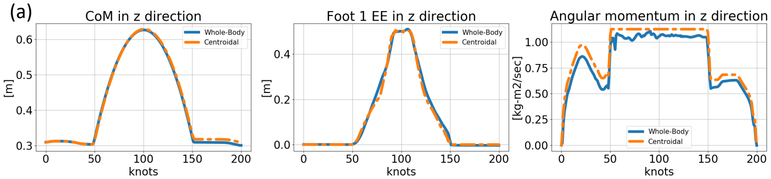
$$\mathbf{w}_h^{k+1} = \mathbf{w}_h^k + \boldsymbol{A}_g(\mathbf{q}^{k+1})\dot{\mathbf{q}}^{k+1} - \begin{bmatrix} m\dot{\mathbf{c}}^{k+1} \\ \mathbf{I}\dot{\boldsymbol{\theta}}^{k+1} \end{bmatrix}$$

$$\mathbf{w}_{ee}^{k+1} = \mathbf{w}_{ee}^k + \text{FK}(\mathbf{q}^{k+1}) - \mathbf{ee}^{k+1}$$

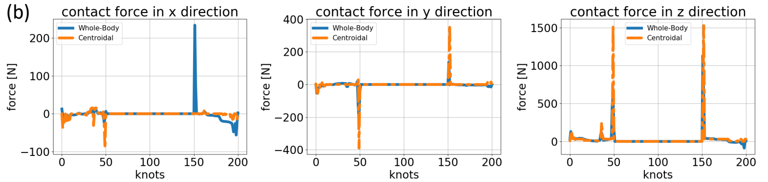
Dual Updates

Results

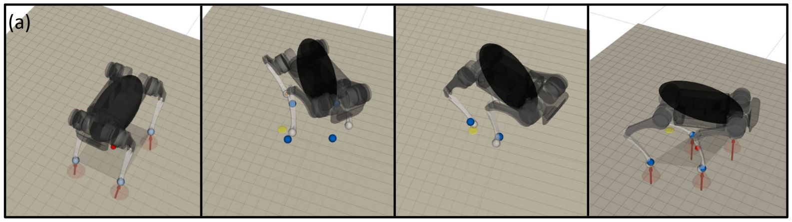
Quadruped Robot jump-twist and trotting examples.

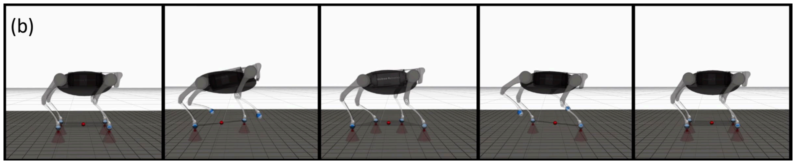


Dynamic consensus of the desired variables for a jump-twist maneuver between centroidal and whole-body models.



Dynamic consensus for a trotting gait motion between centroidal and whole-body models.





Snapshots of an athletic jump-twist maneuver (a) and quadruped trotting gait (b) solved by SNOPT [4] and Crocoddyl [5] for centroidal and whole-body updates respectively.

Discussion and On-going Work

- Designed a centroidal optimization scheme for generating contact sequences and momentum.
- Dynamic **consensus between centroidal and full body** dynamic models.
- On-going work includes improving the angular momentum and inertia tracking. We are also exploring **real-time constrained MPC implementations**. This would require more improvements on the algorithm efficiency and scope for real applications.

References

[1] Z. Zhou and Y. Zhao (2020), [2] A. W. Winkler, C. D. Bellicoso, M. Hutter, and J. Buchli (2018), [3] V. Zordan, D. Brown, A. Macchietto, and K. Yin (2014), [4] P. Gill, W. Murray, and M. Saunders (2005), [5] C. Mastalli, R. Budhiraja, W. Merkt, G. Saurel, B. Hammoud, M. Naveau, J. Carpentier, L. Righetti, S. Vijayakumar and N. Mansard (2020)